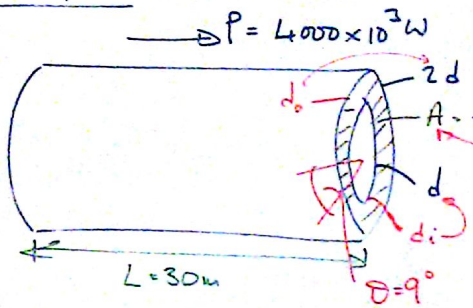
Sheet 4: TORSION + TRANSMISSION.
(Machine Dynamics)

Example 4;

$N = 400$ rpm.

$G = 80 \times 10^9$ N/m²

Find 'd'.



$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$.

$\therefore$ A = c.s.a $= \frac{\pi d_o^2}{4} - \frac{\pi d_i^2}{4}$

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Sheet 4: TORSION + TRANSMISSION.
(Machine Dynamics)

$J_{solid} = \frac{\pi d^4}{32}$

Example 4;

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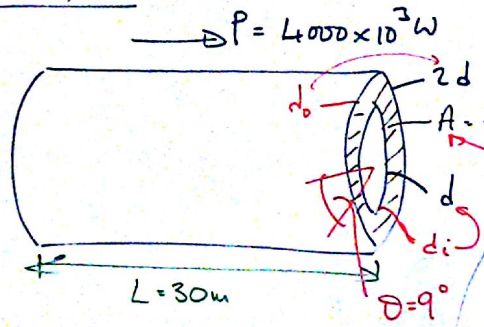
Find 'd'.



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$\therefore$ A = c.s.a $= \frac{\pi d_o^2}{4} - \frac{\pi d_i^2}{4}$

Here, $J_{hollow\ shaft} = \frac{\pi d_o^4}{32} - \frac{\pi d_i^4}{32}$

$= \frac{\pi}{32}\left[d_o^4 - d_i^4\right]$

$J = \frac{\pi}{32}\left[(2d)^4 - (d)^4\right]$

$\therefore J = \frac{\pi}{32}\left[16d^4 - d^4\right]$

$J = \frac{\pi}{32}\left[d^4(16-1)\right]$

$$J = \frac{15\pi d^4}{32} \quad \text{---(1)}$$

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Sheet 4: TORSION + TRANSMISSION.
(Machine Dynamics)

$J_{solid} = \frac{\pi d^4}{32}$

With reference to.

$\frac{T}{J} = \frac{G.\theta}{L} = \frac{\tau}{r}$

Example 4;

$N = 400$ rpm.

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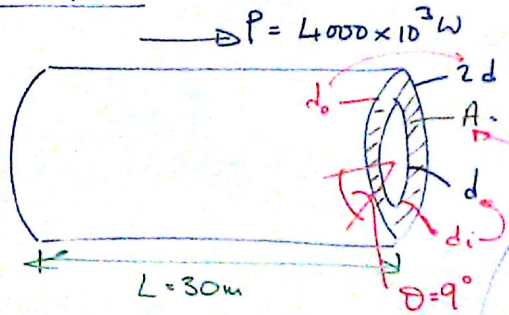

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$J = \frac{\pi}{32}\left[d^4(16-1)\right]$

$$J = \frac{15\pi d^4}{32} \quad \text{---(1)}$$

Here $\theta = 9°$ (convert to rads)

$\therefore 360° = 2\pi$ rad

$1° = \frac{\pi}{180}$ rad

$\therefore 9° = \frac{9\pi}{180}$ rad

$9° = 157.08 \times 10^{-3}$ rad.

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Sheet 4: TORSION + TRANSMISSION.
(Machine Dynamics)

$J_{solid} = \frac{\pi d^4}{32}$

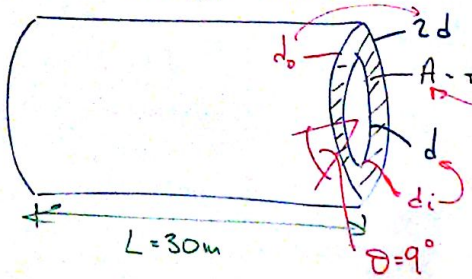
Example 4;

$N = 400 \text{ rpm}$ $\Rightarrow \omega = \frac{400(2\pi)}{60}$

$\omega = 41.89 \text{ rads}^{-1}$

$G = 80 \times 10^9 \text{ N/m}^2$

$P = 4000 \times 10^3 \text{ W}$



Find 'd'.

$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

With reference to.

$$\frac{T}{J} = \frac{G\cdot\theta}{L} = \frac{\tau}{r} \quad (2)$$

Here $\theta = 9^\circ$ (convert to rads)

$\therefore 360^\circ = 2\pi \text{ rad}$

$1^\circ = \frac{\pi}{180} \text{ rad}$

$\therefore 9^\circ = \frac{9\pi}{180} \text{ rad}$

$9^\circ = 157.08 \times 10^{-3} \text{ rad.}$

$P = T.\omega$

$\therefore T = \frac{P}{\omega} = \frac{(4000 \times 10^3)}{(41.89)}$

$$J = \frac{15\pi d^4}{32.} \quad (1)$$

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Sheet 4: TORSION + TRANSMISSION.
(Machine Dynamics)

$J_{solid} = \frac{\pi d^4}{32}$

$P = T.\omega$ $\Rightarrow T = \frac{P}{\omega} = \frac{4000 \times 10^3}{41.89} = 95.49 \times 10^3$ N.m.

$\omega = \frac{400(2\pi)}{60}$

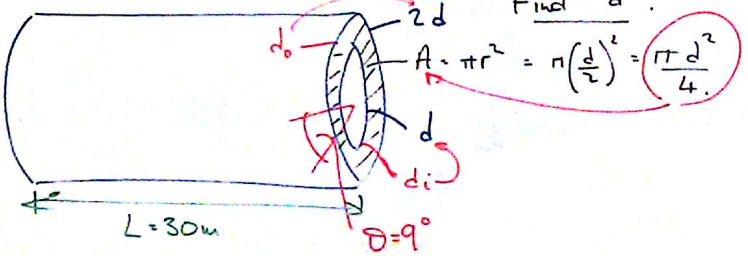
$\omega = 41.89$ rads^{-1}

Example 4;

$N = 400$ rpm

$G = 80 \times 10^9$ N/m^2

$P = 4000 \times 10^3$ W



Find 'd'.

$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

From (2)

$\frac{J}{T} = \frac{L}{G\theta}$

$J = \frac{T.L}{G.\theta}$

Sub for J from eq^n (1)

$\therefore \frac{15\pi d^4}{32} = \frac{T.L}{G.\theta}$

$\therefore d^4 = \frac{32.T.L}{15\pi G.\theta}$

$\therefore d^4 = (32)($

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Sheet 4: TORSION + TRANSMISSION.
(Machine Dynamics)

95.49×10^3 N.m.

$P = T.\omega$. $(T) = \frac{P}{\omega} = \frac{4000 \times 10^3}{41.89}$

$\Rightarrow \omega = \frac{400(2\pi)}{60}$

$\omega = 41.89$ rads^{-1}

$N = 400$ rpm.

$J_{solid} = \frac{\pi d^4}{32}$

Example 4;

$P = 4000 \times 10^3$ W

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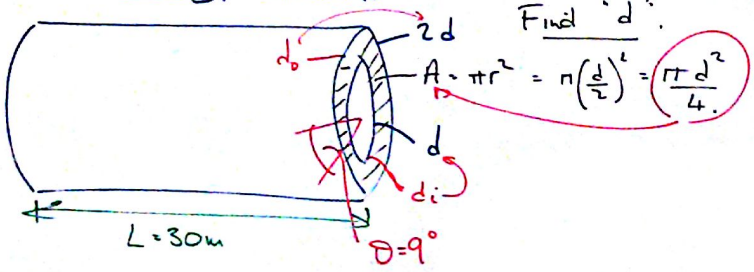
Find 'd'.



From (2)

$\frac{J}{T} = \frac{L}{G\theta}$

$J = \frac{T.L}{G.\theta}$

Sub for J from eq^n (1)

$\therefore \frac{15\pi d^4}{32} = \frac{T.L}{G.\theta}$

$\therefore d^4 = \frac{32.T.L}{15\pi G.\theta}$

$\therefore d^4 = \frac{[(32)(95.49 \times 10^3).(30)]}{[(15).(\pi)(80 \times 10^9).(157.08 \times 10^{-3})]}$

$d^4 = 154.8 \times 10^{-6}$

$\therefore d = \sqrt[4]{154.8 \times 10^{-6}}$

$= (154.8 \times 10^{-6})^{1/4}$

$d = 0.112$ m. = internal Diameter required.

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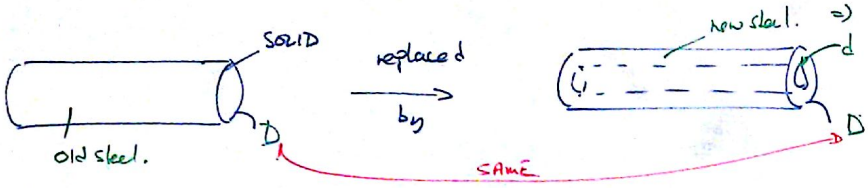
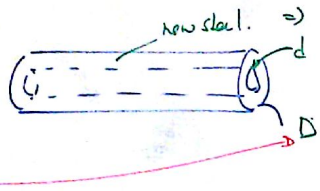
Example 5.

Before weight saving:



$\tau_{max\ old}$.

After weight saving.



$\tau_{max\ new}$ = 40% greater than $\tau_{max\ old}$.

Both shafts have same torsional stiegh G.

Find ratio D : d

To Find ratio D : d.

Initially Let $\tau_{max\ old}$ = Shear stress (max) solid

For new meter $\tau_{max\ new} = (\tau_{max\ old}) + \left(\frac{40}{100} \cdot \tau_{max\ old}\right)$

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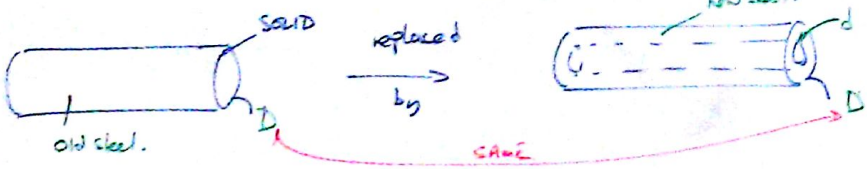
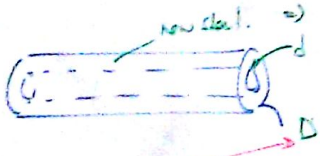
Example 5.

Before weight saving:



$\tau_{max\,old}$.

After weight saving.



$\tau_{max\,new}$ = 40% greater than $\tau_{max\,old}$.

Both shafts have same torsional strength (T).

Find ratio D : d

To Find ratio D : d

Initially let $\tau_{max\,old}$ = shear stress (max) solid

For new material: $\tau_{max\,new} = (\tau_{max\,old}) + \left(\frac{40}{100} \cdot \tau_{max\,old}\right)$

$\therefore \tau_{max\,new} = \tau_{max\,old}(1 + 0.4)$

$\tau_{max\,new} = 1.4\,\tau_{old}$.

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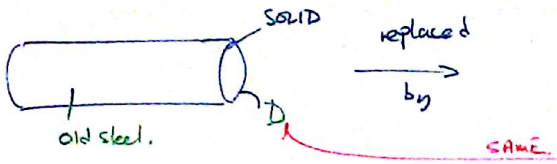
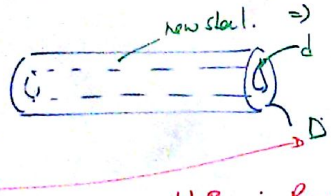
Example 5.

Before weight saving:



$\tau_{max\,old}$.

After weight saving.



$\tau_{max\,new}$ = 40% greater than $\tau_{max\,old}$.

Both shafts have same torsional shear G.

Find ratio D : d

N.B. : Rigidity remains constant, before & after

$\therefore G$ = constant.

$\therefore G_{old} = G_{new}$.

To find ratio D : d.

Initially let $\tau_{max\,old}$ = shear stress (max) solid

For new material: $\tau_{max\,new} = (\tau_{max\,old}) + \left(\frac{40}{100} \cdot \tau_{max\,old}\right)$

$\therefore \tau_{max\,new} = \tau_{max\,old}(1 + 0.4)$

$\tau_{max\,new} = 1.4\,\tau_{old}$.

we know:

$$\frac{\tau}{J} = \frac{G \cdot \theta}{L} = \frac{\tau}{r} \quad \text{— (A)}$$

$$G = \frac{\tau \cdot L}{\theta \cdot r} = \frac{\tau \cdot L}{\theta\left[\frac{d}{2}\right]} = \frac{\tau \cdot L}{\left[\frac{\theta \cdot d}{2}\right]}$$

$$G = \frac{2\tau L}{\theta \cdot d}$$

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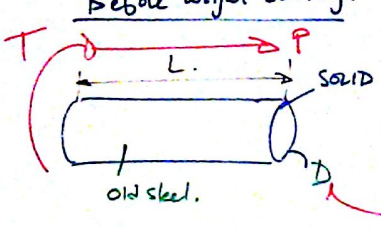
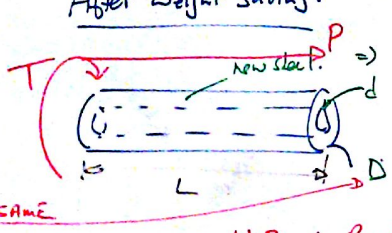
Example 5.

Before weight saving:



$\tau_{max\,old}$.

After weight saving.



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τ_{max} (NEW) = 40% greater than $\tau_{max\,old}$.

Both shafts have same torsional shear G.

Find ratio D : d

N.B. : Rigidity remains constant, before & after

$\therefore G$ = constant.

$\therefore G_{old} = G_{new}$.

Before.

$$G = \frac{2\tau L}{\theta . d}$$

let = ' N.B.

$$G = \frac{2\tau}{\theta . d}$$

we know;

$$\frac{T}{J} = \frac{G.\theta}{L} = \frac{\tau}{r} \quad (A)$$

Use;

$$\frac{T}{J} = \frac{G.\theta}{L} \Rightarrow \boxed{G = \frac{T.L}{J.\theta}}$$

$$G = \frac{\tau . L}{\theta . r} = \frac{\tau . L}{\theta . \left[\frac{d}{2}\right]} = \frac{\tau . L}{\left[\frac{\theta . d}{2}\right]}$$

$$\boxed{G = \frac{2\tau L}{\theta . d}}$$

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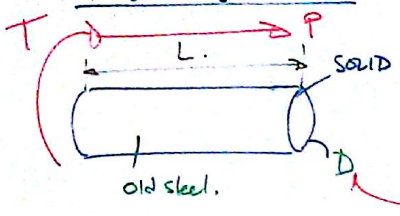
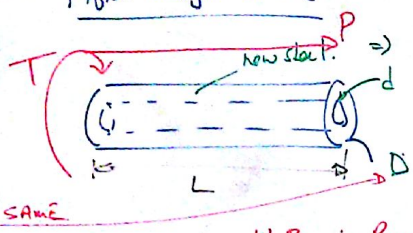


$\tau_{max\,old}$.

After weight saving.



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$\tau_{max\,new}$ = 40% greater than $\tau_{max\,old}$.

Both shafts have same torsional shaft G.

Find ratio $D : d$

N.B. : Rigidity remains constant, before & after

$\therefore G$ = constant.

$\therefore G_{old} = G_{new}$.

N.B. T, L, θ are constant.

$\therefore$ let $\left[\frac{T.L}{\theta} = k\right]$

$\therefore G = k \cdot \frac{1}{J}$.

$\therefore \boxed{G = \frac{k}{J}}$ — (P)

Before: $G = \frac{2\tau L}{\theta . d}$, let $k = 1$

$\boxed{G = \frac{2}{\theta . d}}$

Use: $\frac{T}{J} = \frac{G.\theta}{L} \Rightarrow \boxed{G = \frac{T.L}{J.\theta}}$

we know:

$$\boxed{\frac{T}{J} = \frac{G.\theta}{L} = \frac{\tau}{r}} \quad \text{— (A)}$$

$$G = \frac{\tau . L}{\theta . r} = \frac{\tau . L}{\theta \left[\frac{d}{2}\right]} = \frac{\tau . L}{\left[\frac{\theta . d}{2}\right]}$$

$$\boxed{G = \frac{2\tau L}{\theta . d}}$$

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Example 5.

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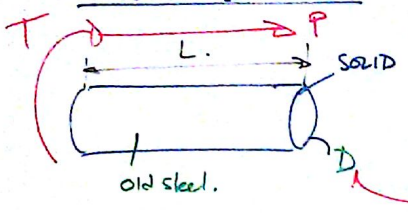

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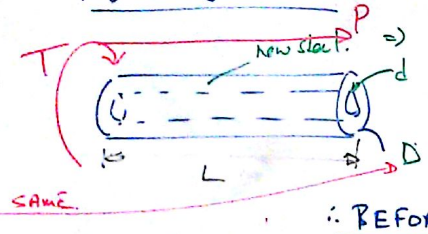
After weight saving.



SAME.

τ_{max} old.

τ_{max} new = 40% greater than τ_{max} old.

Both shafts have same torsional stiffness G.

Find ratio D : d

∴ BEFORE: SOLID. ⇒ $J = \frac{\pi D^4}{32}$

$\therefore G_{solid} = \frac{k}{\left[\frac{\pi D^4}{32}\right]}$

$\therefore \boxed{G_{solid} = \frac{32k}{\pi D^4}}$ — (α)

Before: $G = \frac{2\tau L}{\theta . d}$ let = k

$\therefore \boxed{G = \frac{2\tau}{\theta . d}}$

N.P. T, L, θ are con

$\therefore$ let $\boxed{\frac{T.L}{\theta} = k}$

$\therefore G = k . \frac{1}{J}$

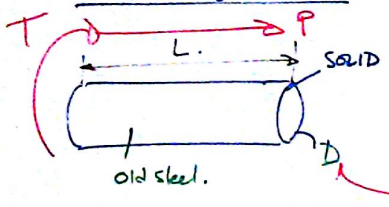
$\therefore \boxed{G = \frac{k}{J}}$ — (β)

Use: $\frac{T}{J} = \frac{G.\theta}{L} \Rightarrow \boxed{G = \frac{T.L}{J.\theta}}$

NB. use (β) Before and after then equate. For "G"

Example 5.

Before weight saving:



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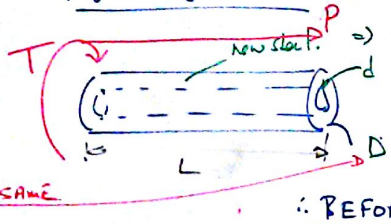
After weight saving.



SAME

τ_{max} new = 40% greater than τ_{max} old.

Both shafts have same torsional stiffness G.

Find ratio D : d

∴ BEFORE: SOLID. $\Rightarrow J = \frac{\pi D^4}{32}$

$\therefore G_{solid} = \frac{k}{\left[\frac{\pi D^4}{32}\right]}$

18 mm dia

After: HOLLOW SHAFT.

$J_{new} = \frac{\pi}{32}\left[D^4 - d^4\right]$

$\therefore G_{Hollow} = \frac{k}{\frac{\pi(D^4 - d^4)}{32}}$

$\therefore G_{Hollow} = \frac{32k}{\pi(D^4 - d^4)}$

The same G value.

∴ EQUATE.

$\therefore \frac{32k}{\pi D^4} = \frac{32k}{\pi(D^4 - d^4)}$

$\Rightarrow D^4 = D^4 - d^4$

$\Rightarrow D^4 - D^4 = -d^4$ ×

$\therefore G_{solid} = \frac{32k}{\pi D^4}$ — (A)

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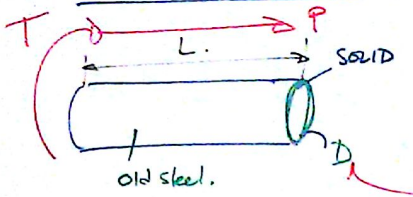
Example 5.

Before weight saving:



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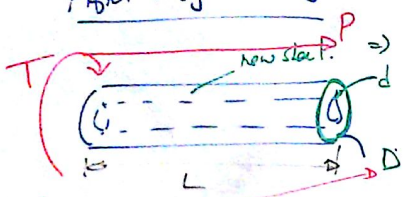
After weight saving.



τ_{max} new = 40% greater than τ_{max} old.

Both shafts have same torsional stiffness G_T.

Find ratio D : d

Ref: $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$ — (1)

$J_1 = \frac{32k}{\pi D^4}$ $\quad J_2 = \frac{32k}{\pi(D^4 - d^4)}$

Ref: $\frac{T}{J} = \frac{\tau}{r}$ — (2)

Solid shaft

From (2)

Before: $\frac{T}{J_{old}} = \frac{\tau}{\left(\frac{D}{2}\right)}$ $\quad \therefore \frac{D}{\tau} = \frac{J_{old}}{T}$

$\therefore D = \frac{\tau J_{old}}{T}$ — (3)

Contact — old + new

$\Rightarrow D_{old} = D_{new} = D$

$D_{old} = \frac{\tau}{T} \cdot \frac{32k}{\pi D^4}$

$\therefore$

$D_{new} = \frac{\tau}{T} \cdot J_{new}$

$D_{new} = \frac{\tau}{T} \cdot \frac{32k}{\pi(D^4 - d^4)}$

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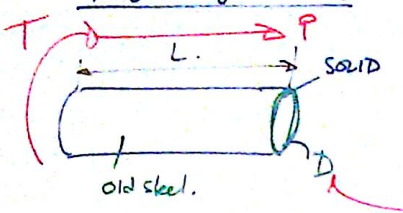
Example 5.

Before weight saving:



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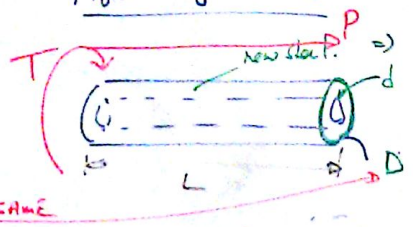
After weight saving.



τ_{max} new = 40% greater than τ_{max} old.

Both shafts have same torsional shear G.

Find ratio D : d

Reb: $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$ — Eqn (2)

$J_1 = \frac{32k}{\pi D^4}$ $\quad J_2 = \frac{32k}{\pi(D^4 - d^4)}$

Before: $\frac{T}{J_{old}} = \frac{\tau}{\left(\frac{D}{2}\right)}$ $\quad \therefore \frac{D}{\tau} = \frac{J_{old}}{T}$

$\therefore D = \frac{\tau J_{old}}{T}$ — (3)

$\frac{T}{J_{old}} = \frac{2\tau}{D}$

$\frac{J_{old}}{T} = \frac{D}{2\tau}$ $\Rightarrow$ $\frac{J_{new}}{T} = \frac{D}{2\tau}$

Constant — old + new.

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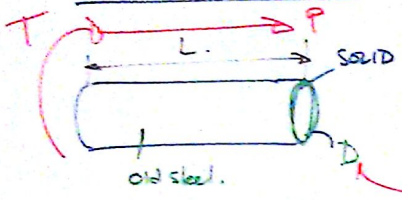
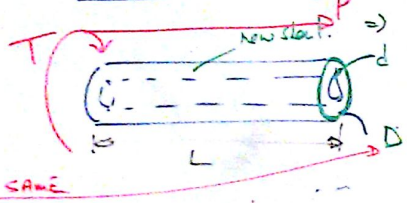
Example 5.

Before weight saving:



replaced by

After weight saving.



SAME

τ_{max} (new) = 40% greater than τ_{max} (old).

Both shafts have same torsional shear $G\tau$.

Find ratio $D : d$

Ref: $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$

$J_1 = \frac{32k}{\pi D^4}$ $\quad J_2 = \frac{32k}{\pi(D^4 - d^4)}$

From (2)

Before: $\frac{T}{J_{old}} = \frac{\tau}{\left(\frac{D}{2}\right)}$ $\quad \therefore \frac{D}{\tau} = \frac{J_{old}}{T}$

$\frac{T}{J_{old}} = \frac{2\tau}{D}$

$\therefore D = \frac{\tau J_{old}}{T}$ (3)

$\frac{J_{old}}{T} = \frac{D}{2\tau}$ $\Rightarrow$ $\frac{J_{new}}{T} = \frac{D}{2\tau}$

(constant) (old + new)

$\therefore D = \frac{J_{old} \cdot 2\tau}{T} = \frac{2\tau \cdot J_{new}}{T}$

$J_{old} = J_{new} \Rightarrow \frac{32k}{\pi D^4} = \frac{32k}{\pi(D^4 - d^4)}$ $\Rightarrow \frac{D^4}{D^4 - d^4} = 1$

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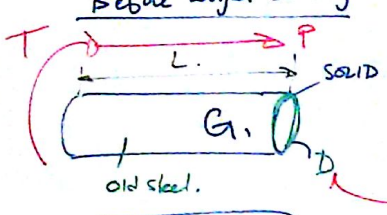
Example 5.

Before weight saving:



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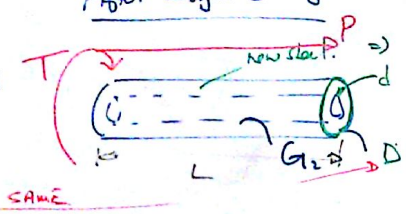
After weight saving.



τ_{max} new = 40% greater than τ_{max} old.

$\therefore \boxed{G_1 \neq G_2}$ since $G = E = \frac{\sigma}{\epsilon}$

$\frac{\tau}{\gamma}$ — Different.

H.B. $G = \frac{\tau}{\gamma}$ where $\boxed{\tau_2 = 1.4\,\tau_0}$

$$\boxed{\frac{T}{J}} = \frac{G\theta}{L} = \boxed{\frac{\tau}{r}}$$

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$$\frac{T}{J} = \frac{\tau}{r}$$

$T.r = J.\tau$

$$\tau_2 = 1.4\,\tau_0$$

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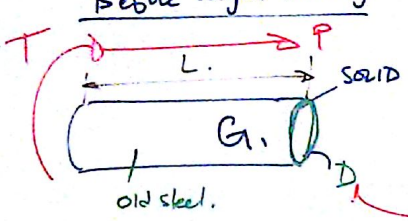
Example 5.

Before weight saving:



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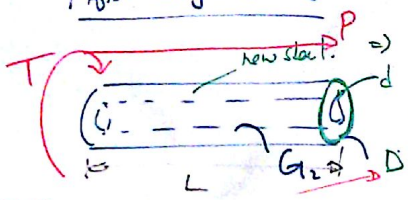

τ_{max} (NW) = 40% greater than τ_{max} old.

SAME

old steel.

TRY: $\frac{T}{J} = \frac{\tau}{r}$

Before.

$J_{old} = \frac{\pi D^4}{32}$

$J_{new} = \frac{\pi}{32}(D^4 - d^4)$

$\therefore$ old steel (solid)

$\frac{T}{\left(\frac{\pi D^4}{32}\right)} = \frac{\tau_0}{\left(\frac{D}{2}\right)}$

$\frac{32T}{\pi D^4} = \frac{2\tau_0}{D}$ (x)

new steel

New steel (Hollow shaft).

$\frac{T}{\left[\frac{\pi}{32}(D^4 - d^4)\right]} = \frac{1.4\tau_0}{\frac{D}{2}}$

$= \frac{32T}{\pi(D^4 - d^4)} = \frac{2.8\tau_0}{D}$

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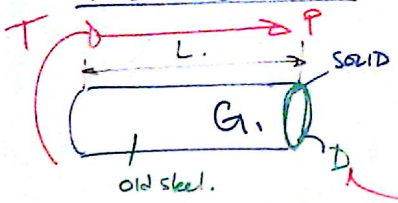
Example 5.

Before weight saving:



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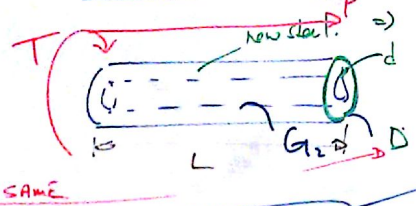

$\Rightarrow$ τ_{max} (new) = 40% greater than τ_{max} (old).

SAME

old steel.

$$\frac{T}{J} = \frac{\tau}{r}$$

new steel

Now steel (Hollow shaft).

Eqn x.

$$\tau_0 = \frac{16}{\pi} \frac{2T}{D^4} D$$

$$\tau_0 = \frac{16\,TD}{\pi D^4}$$

$$\tau_0 = \frac{16T}{\pi D^3}$$

(4)

$\therefore$ old steel (solid)

$$\frac{T}{\left(\frac{\pi D^4}{32}\right)} = \frac{\tau_0}{\left(\frac{D}{2}\right)}$$

$$\frac{32T}{\pi D^4} = \frac{2\tau_0}{D}$$ (x)

$$\frac{T}{\left[\frac{\pi}{32}(D^4 - d^4)\right]} = \frac{1.4\tau_0}{\frac{D}{2}}$$

$$= \frac{32T}{\pi(D^4 - d^4)} = \frac{2.8\tau_0}{D}$$ (y)

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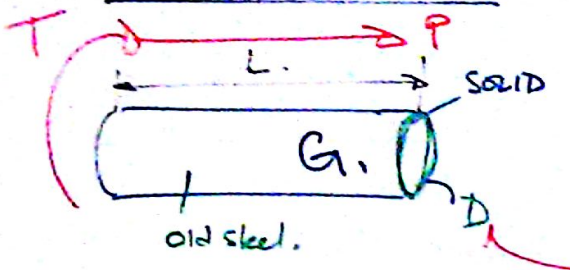
Example 5.

Before weight saving:



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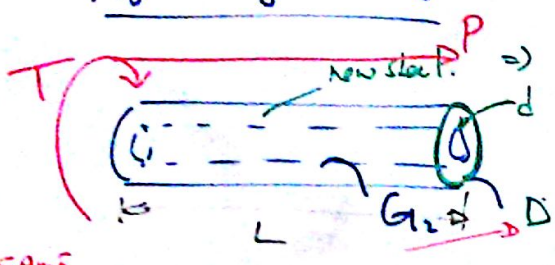
After weight saving.



SAME

$\Rightarrow$ τ_{max} (NW) = 40% greater than τ_{max} old.

From x.

$$\tau_0 = \frac{16}{\pi D^3}\,T$$

From (4)

$$\tau_0 = \frac{32TD}{2.8\pi(D^4 - d^4)}$$

equate.

$$\Rightarrow \frac{16T}{\pi D^3} = \frac{32TD}{2.8\pi(D^4 - d^4)}$$

$$\frac{T}{D^3} = \frac{2TD}{2.8(D^4 - d^4)}$$

$$\frac{1}{D^3} =$$

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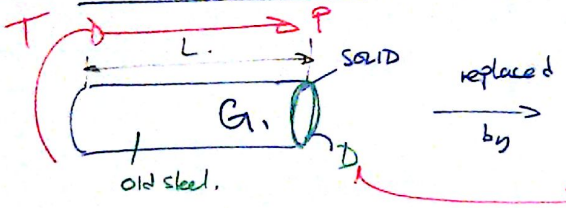
Example 5.

Before weight saving:



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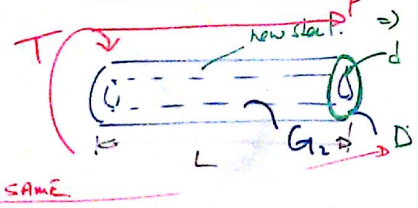

SAME

$\Rightarrow \tau_{max}$ (HW) = 40% greater than τ_{max} (old).

From x.

$$\tau_o = \frac{16}{\pi} \frac{2TD}{2\pi D^4}$$

$$\tau_o = \frac{16\,TD}{\pi D^4}$$

$$\tau_o = \frac{16T}{\pi D^3} \quad (4)$$

From (4)

$$\tau_o = \frac{32TD}{2.8\pi(D^4 - d^4)}$$

equate.

$$\Rightarrow \frac{16T}{\pi D^3} = \frac{32TD}{2.8\pi(D^4 - d^4)}$$

$$\frac{T}{D^3} = \frac{2TD}{2.8(D^4 - d^4)}$$

$$\frac{1}{D^3} = \frac{2D}{2.8(D^4 - d^4)}$$

$$1 = \frac{2D^4}{2.8(D^4 - d^4)}$$

or $2.8(D^4 - d^4) = 2D^4$

$\therefore 2.8D^4 - 2.8d^4 = 2D^4$

$\therefore 2.8D^4 - 2D^4 = 2.8d^4$

$\therefore D^4(2.8 - 2) = 2.8d^4$

$0.8D^4 = 2.8d^4$

$\therefore \frac{D^4}{d^4} = \frac{2.8}{0.8} = 3.5.$

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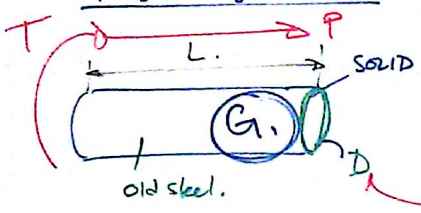
Example 5.

Before weight saving:



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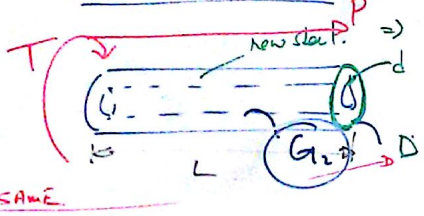

SAME.

τ_{max} (HW) = 40% greater than τ_{max} old.

$G = \frac{\sigma}{\epsilon}$ $\quad \tau$ $\quad \gamma$

From x.

$$\tau_0 = \frac{16}{\pi D^3}$$

$$\tau_0 = \frac{16 T D}{\pi D^4}$$

$$\tau_0 = \frac{16T}{\pi D^3}$$

(4)

From (4)

$$\tau_0 = \frac{32TD}{2.8\pi(D^4 - d^4)}$$

equate.

$$\Rightarrow \frac{16T}{\pi D^3} = \frac{32TD}{2.8\pi(D^4 - d^4)}$$

$$\frac{T}{D^3} = \frac{2TD}{2.8(D^4 - d^4)}$$

$$\frac{1}{D^3} = \frac{2D}{2.8(D^4 - d^4)}$$

$$1 = \frac{2D^4}{2.8(D^4 - d^4)}$$

$$\text{or } 2.8(D^4 - d^4) = 2D^4$$

$$\therefore 2.8D^4 - 2.8d^4 = 2D^4$$

$$\therefore 2.8D^4 - 2D^4 = 2.8d^4$$

$$\therefore D^4(2.8 - 2) = 2.8d^4$$

$$0.8D^4 = 2.8d^4$$

$$\therefore \frac{D^4}{d^4} = \frac{2.8}{0.8} = 3.5.$$

$$\therefore D : d = 3.5 : 1.$$

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Example 5.

Before weight saving:

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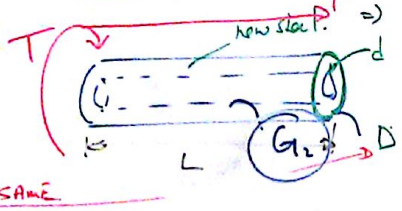
After weight saving.



SAME

τ_{max} in NW = 40% greater than τ_{max} in SOLID.

$G = \frac{\sigma}{\epsilon}$ $\quad \tau$ $\quad \gamma$.

From X.

$$\tau_0 = \frac{16}{\pi D^3}$$

From Y

$$\tau_0 = \frac{32TD}{2.8\pi(D^4 - d^4)}$$

equate.

$$\Rightarrow \frac{16T}{\pi D^3} = \frac{32TD}{2.8\pi(D^4 - d^4)}$$

$$\frac{1}{D^3} = \frac{2D}{2.8(D^4 - d^4)}$$

$$\therefore D : d = 3.5 : 1.$$

$$1 = \frac{2D^4}{2.8(D^4 - d^4)}$$

$$\text{or } 2.8(D^4 - d^4) = 2D^4$$

$$\therefore 2.8D^4 - 2.8d^4 = 2D^4$$

$$\therefore 2.8D^4 - 2D^4 = 2.8d^4$$

$$\therefore D^4(2.8 - 2) = 2.8d^4$$

$$0.8D^4 = 2.8d^4$$

$$\therefore \frac{D^4}{d^4} = \frac{2.8}{0.8} = 3.5.$$

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